Stats 2 Discrete Random Variables Questions

5 The Globe Express agency organises trips to the theatre. The cost, £X, of these trips can be modelled by the following probability distribution:

x	40	45	55	74
$\mathbf{P}(X=x)$	0.30	0.24	0.36	0.10

(a) Calculate the mean and standard deviation of X. *(4 marks)*

(b) For special celebrity charity performances, Globe Express increases the cost of the trips to £Y, where

$$Y = 10X + 250$$

Determine the mean and standard deviation of Y. *(2 marks)*

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3 Morecrest football team always scores at least one goal but never scores more than four goals in each game. The number of goals, R, scored in each game by the team can be modelled by the following probability distribution.

r	1	2	3	4
$\mathbf{P}(R=r)$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

(a) Calculate exact values for the mean and variance of R. *(4 marks)*

(b) Next season the team will play 32 games. They expect to win 90% of the games in which they score at least three goals, half of the games in which they score exactly two goals and 20% of the games in which they score exactly one goal.

Find, for next season:

(i) the number of games in which they expect to score at least three goals; *(1 mark)*

(ii) the number of games that they expect to win. *(2 marks)*

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4 The number of fish, X, caught by Pearl when she goes fishing can be modelled by the following discrete probability distribution:

x	1	2	3	4	5	6	$\geqslant 7$
$\mathbf{P}(X = x)$	0.01	0.05	0.14	0.30	k	0.12	0

(a) Find the value of k. *(1 mark)*

(b) Find:

(i) $\mathrm{E}(X)$; *(1 mark)*

(ii) $\mathrm{Var}(X)$. *(3 marks)*

(c) When Pearl sells her fish, she earns a profit, in pounds, given by

$$Y = 5X + 2$$

Find:

(i) $\mathrm{E}(Y)$; *(1 mark)*

(ii) the standard deviation of Y. *(3 marks)*

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7 On a multiple choice examination paper, each question has five alternative answers given, only one of which is correct. For each question, candidates gain 4 marks for a correct answer but lose 1 mark for an incorrect answer.

(a) James guesses the answer to each question.

(i) Copy and complete the following table for the probability distribution of X, the number of marks obtained by James for each question.

x	4	-1
$\mathbf{P}(X=x)$		

(1 mark)

(ii) Hence find $\mathrm{E}(X)$. *(2 marks)*

(b) Karen is able to eliminate two of the incorrect answers from the five alternative answers given for each question before guessing the answer from those remaining.

Given that the examination paper contains 24 questions, calculate Karen's expected total mark. *(4 marks)*

Stats 2 Discrete Random Variables Answers

Question	Solution	Marks	Total	Comments
5(a)	$\mathrm{E}(X)=\sum_{\text{all } x} x\ \mathrm{P}(X=x)$			
	$=50$	B1		(cao)
	$\mathrm{E}(X^2)=\sum_{\text{all } x} x^2\mathrm{P}(X=x)$			
	$=2602.6(0)$	M1		
	$\mathrm{Var}(X)=\mathrm{E}(X^2)-[\mathrm{E}(X)]^2$			
	$=2602.6-50^2$	M1		
	$=102.6(0)$			
	$\Rightarrow$ standard deviation $(X)=10.13$	A1	4	(to nearest 1p)
(b)	$\mathrm{E}(Y)=\mu=\mathrm{E}(10X+250)$			
	$=10\times\mathrm{E}(X)+250$			
	$=750$	B1√		(on their $\mathrm{E}(X)$)
	$\mathrm{s.d}(Y)=10\times10.1$			
	$=101$	B1√	2	(on their $\mathrm{sd}(X)$)
	Total		**6**	

Q	Solution	Marks	Total	Comments
3(a)	$\mathrm{E}(R)=\sum_{\text{all } r} r\ \mathrm{P}(R=r)$			
	$=\left(1\times\frac{7}{16}\right)+\left(2\times\frac{5}{16}\right)+\left(3\times\frac{3}{16}\right)+\left(4\times\frac{1}{16}\right)$			
	$=\frac{30}{16}$			
	$=1\frac{7}{8}$	B1		(1.875)
	$\mathrm{E}(R^2)=\sum_{\text{all } r} r^2\ \mathrm{P}(R=r)$			
	$=\frac{70}{16}$ or $4\frac{3}{8}$	B1		(4.375)
	$\mathrm{Var}(R)=4\frac{3}{8}-\left(1\frac{7}{8}\right)^2$	M1		

	$= \frac{220}{256}$ or $\frac{55}{64}$	A1	4	(0.859375)
(b)(i)	$32 \times \frac{1}{4} = 8$	B1	1	
(ii)	$= \left(32 \times \frac{7}{16} \times \frac{1}{5}\right) + \left(32 \times \frac{5}{16} \times \frac{1}{2}\right) + 8 \times \frac{9}{10}$	M1		
	$= 2.8 + 5 + 7.2$			A0 if these numbers rounded before adding
	$= 15$	A1	2	
	Total		**7**	

4(a)	$\sum p = 1$ $k = 1 - (0.01 + 0.05 + 0.14 + 0.30 + 0.12)$ $k = 0.38$	B1	1	
(b)(i)	$\mathrm{E}(X) = \sum_{\text{all } x} x \, \mathrm{P}(X = x) = 4.35$	B1	1	$\frac{87}{20}$
(ii)	$\mathrm{Var}(X) = \sum_{\text{all } x} x^2 \, \mathrm{P}(X = x) - \mu^2$	M1		$\mathrm{E}(X^2)$ attempted
	$= 20.09 - 18.9225$	M1		$\sum x^2 \mathrm{P}(X = x) - \mu^2$
	$= 1.1675$	A1	3	$\frac{467}{400}$ (AWFW 1.16 – 1.17)
(c)(i)	$\mathrm{E}(Y) = 5\mathrm{E}(X) + 2$ $= 5 \times 4.35 + 2$ $= 23.75$	M1	1	Their (b)(i) × 5 + 2
(ii)	$\mathrm{Var}(Y) = 25\mathrm{Var}(X)$ $= 29.1875$	M1		Their (b)(ii) × 25
	Standard deviation $= 5.40$	m1 A1	3	$\sqrt{}$ $(5.40 - 5.41)$
	Total		**9**	

Q	Solution	Marks	Total	Comments
7(a)(i)	$\begin{array}{\|c\|c\|c\|}\hline x & 4 & -1 \\ \hline \mathbf{P}(X=x) & \frac{1}{5} & \frac{4}{5} \\ \hline \end{array}$	B1	1	
(ii)	$\mathrm{E}(X)=\left(4\times\frac{1}{5}\right)+\left(-1\times\frac{4}{5}\right)=0$	M1 A1	2	$\left(p>0, \sum p=1\right)$
(b)	$\begin{array}{\|c\|c\|c\|}\hline x & 4 & -1 \\ \hline \mathbf{P}(X=x) & \frac{1}{3} & \frac{2}{3} \\ \hline \end{array}$	B1		
	$\mathrm{E}(X)=\left(4\times\frac{1}{3}\right)+\left(-1\times\frac{2}{3}\right)=\frac{2}{3}$	B1		$\left(p>0, \sum p=1\right)$
	$\mathrm{E}(24X)=24\times\mathrm{E}(X)$			
	$=24\times\frac{2}{3}$	M1		
	$=16$	A1	4	
	Total		**7**	